FD-612

M.A./M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - I

Integration Theory and Functional Analysis

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) If E is a measurable set of finite positive measure, i. e., $0 < \nu(E) < \infty$, then prove that E contains a positive set A with $\nu(A) > 0$.

(*b*) State and prove Lebesgue Decomposition theorem.

(*c*) State and prove Caratheodory Extension theorem.

Unit-II

2. (*a*) If A is a μ-measurable subset of X and B is a ν-measurable subset of Y, then prove that $A \times B$ is a $\mu \times \nu$-measurable subset of $X \times Y$.

(*b*) Let E be a set in $R_{\sigma\delta}$ with $(\mu \times \nu)(E) < \infty$. Then show that the function g defined by

$$g(x) = \nu(E_x)$$

is a measurable function of x and $\int g \, d\mu = (\mu \times \nu)(E)$.

(*c*) Prove that every finite signed Borel measure μ on R^k that is absolutely continuous with respect to the Lebesgue measure λ, is differentiable almost everywhere.

Unit-III

3. (*a*) Prove that every compact Baire set is a G_δ.

(*b*) Let μ be a measure defined on a σ-algebra }}} containing the Baire sets. If μ is quasi regular, then prove that for each $E \in$ }}} with $\mu(E) < \infty$ there is a Baire set B with

$$\mu(E \,\Delta\, B) = 0$$

(*c*) State and prove Riesz-Markoff theorem.

Unit-IV

4. (*a*) Let X be a non-zero finite-dimensional linear space of dimension n. If X is complete, then show that it is isomorphic to C^n.

(*b*) Show that on a finite dimensional linear space all norms are equivalent.

(*c*) Show that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

Unit-V

5. (*a*) Prove that in a normed linear space X, $x_n \xrightarrow{w} x$ if and only if :

(*i*) The sequence $\{\| x_n \|\}$ is bounded.

(*ii*) For every element f of a total subset $M \subset X^*$, $f(x_n) \to f(x)$.

(*b*) Let X and Y be normed linear spaces and T a linear transformation on X into Y. Then T is continuous either at every point of X or at no point of X. It is continuous on X if and only if there is a constant M such that $\| Tx \| \leq M \cdot \| x \|$ for every x in X.

(*c*) Show that the dual space of c_0 is l_1.

FD-613

M.A/M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Paper - II

Partial Differential Equations
and Mechanics - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Solve the partial differential equation $p + r + s = 1$

(*b*) If ϕ is harmonic function in R_1 and $\frac{\partial \phi}{\partial n} = 0$ on R_2, then ϕ is a constant in $\bar{R}$.

(*c*) Find the Green's function for the Dirichlet problem on the rectangle $R_1 : 0 \le x \le a,\ 0 \le y \le b$, described by the PDE.

$$(\Delta^2 + \lambda)u = 0 \text{ in } R_1$$

and the BC, $u = 0$ on R_2

Unit-II

2. (*a*) State and prove Mean value theorem for Harmonic function.

(*b*) Derive the one dimensional wave equation.

(*c*) Obtain the solution of the heat flow equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.

Unit-III

3. (*a*) State and prove Lagrange's equation of first kind.

(*b*) Derive the Hamilton canonical equations.

(*c*) Derive Ruth's equation.

Unit-IV

4. (*a*) Define Poisson bracket. If $[\phi, \psi]$ be the Poisson bracket of ϕ and ψ, then prove that :

(*i*) $$\frac{\partial}{\partial t}[\phi, \psi] = \left[\frac{\partial \phi}{\partial t}, \psi\right] + \left[\phi, \frac{\partial \psi}{\partial t}\right]$$

(*ii*) $$\frac{d}{dt}[\phi, \psi] = \left[\frac{d\phi}{dt}, \psi\right] + \left[\phi, \frac{d\psi}{dt}\right]$$

(*b*) Find a curve joining two points along with a particle falling from rest under the influence of gravity travels from higher to the lower point in the minimum time.

(*c*) Show that the transformation :

$$P = \frac{1}{2}\left(p^2 + q^2\right), \quad Q = \tan^{-1}\left(\frac{q}{p}\right)$$

is canonical.

Unit-V

5. (*a*) Find the attraction of thin spherical shell of mean M and radius a.

(*b*) Show that the potential of a uniform spherical shell, of small thickness *k*, density ρ and radius *a* at an external point-distant *c* from the centre is

$$\frac{2\pi\gamma k\rho a}{(n+1)(n+3)c}\left[(c+a)^{n+2}-(c-a)^{n+3}\right]$$

(*c*) State and prove Gauss' theorem.

FD-616

M.A./M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Optional (C)

Paper - III

Fuzzy Sets and its Applications - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Define law of excluded middle and law of contradiction and discuss the distributive property of (i, u, c) which satisfies these two laws.

(*b*) State characterization theorem of t-conorms and find t-conorm for $g(a) = 1 - (1 - a)^p$.

(*c*) Define convexity for a set graphically and show that a Fuzzy set A on R is convex iff

$A\,(\lambda x_1 + (1-\lambda)x_2) \geq \min\,[A(x_1)\,,\, A(x_2)].$

2. (*a*) Explain extension principle, how it differs from crisp function. Show that $\alpha\,[\,f(A)] \geq f(\alpha_A)$. Give a supportive example.

(*b*) Solve Fuzzy equation $A + X = B$ where

$$A = \frac{\cdot 3}{[0,1)} + \frac{\cdot 5}{[1,2)} + \frac{\cdot 8}{[2,3)} + \frac{\cdot 9}{[3,4)} + \frac{1}{4} + \frac{\cdot 6}{(4,5]} + \frac{\cdot 2}{(5,6]}$$

$$B = \frac{\cdot 2}{[0,1)} + \frac{\cdot 3}{[1,2)} + \frac{\cdot 6}{[2,3)} + \frac{\cdot 5}{[3,4)} + \frac{\cdot 8}{[4,5)} + \frac{1}{6} + \frac{\cdot 5}{(6,7]} + \frac{\cdot 4}{(7,8]} + \frac{\cdot 2}{(8,9]} + \frac{\cdot 1}{(9,10]}$$

(*c*) $$A(x) = \begin{cases} 0 & \text{for } x < -2 \text{ and } x > 4 \\ \dfrac{x+2}{3} & \text{for } -2 \leq x \leq 1 \\ \dfrac{4-x}{3} & \text{for } 1 \leq x \leq 4 \end{cases}$$

$$B(x)=\begin{cases}0 & \text{for } x<1 \text{ and } x>3\\ x-1 & \text{for } 1\le x\le 2\\ 3-x & \text{for } 2\le x\le 3\end{cases}$$

Find
MIN $(A, B)(x)$ and MAX $(A, B)(x)$.

3. (*a*) Define crisp and fuzzy relations. Let $X = \{1, 2, \ldots.., 10\}$. The cartesian product $(x \times y)$ contains 100 members. Let $R(X, X) = \{(x, y) \mid x$ and y have the same remainder when divided by $3\}$. Is R an equivalance relation on X? Find equivalance classes.

(*b*) Write a short note on Fuzzy morphisms.

(*c*) Prove that

(*i*) $w_i(a, d) \ge b$ iff $i(a, b) \le d$

(*ii*) $w_i(\inf a_j, b) \ge \sup w_i(a_j, b)$

4. (*a*) Let $X = \{1, 2, \ldots., 100\}$, $Y = \{50, 51, \ldots.,100\}$

$$R(X,Y)=\begin{cases}1-\dfrac{x}{y} & x\le y\\ 0 & \text{otherwise}\end{cases}$$

(*i*) What is the domain of R?

(*ii*) What is the range of R?

(*iii*) Calculate R^{-1}

(*b*) Prove that min join are associative operations on binary fuzzy relations.

(*c*) Write a short note on fuzzy compatibility relations.

5. (*a*) Define the following :

(*i*) Total ignorance

(*ii*) Fuzzy measure

(*iii*) Degree of belief

(*iv*) Necessity measure

(*b*) If $X = \{a, b, c, d\}$, $m_1(a, b) = \cdot 2$, $m_1(a, c) = \cdot 3$, $m_1(b, d) = \cdot 5$, $m_2(a, d) = \cdot 2$, $m_2(b, c) = \cdot 5$, $m_2(a, b, c) = \cdot 3$. Calculate the basic probability assignment.

(*c*) $F = \frac{\cdot 4}{1} + \frac{\cdot 7}{2} + \frac{1}{3} + \frac{\cdot 8}{4} + \frac{\cdot 5}{5}$ and $A(x) = 0$ for all $x \notin \{1, 2, 3, 4, 5\}$. Determine Nec ($A$) and Pos ($A$).

FD-618

M.A./M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Optional - A

Paper - IV

Operations Research - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Solve the following linear programming problem by simplex method :

Maximize $Z_1 = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

(*b*) Apply the principle of duality to solve the linear programming problem :

Maximize $Z_1 = 3x_1 - 2x_2$

Subject to $x_1 + x_2 \leq 5$

$x_1 \leq 4$

$1 \leq x_2 \leq 6$ and $x_1, \ x_2 \geq 0$

(*c*) A Steel company manufactures three products P_1, P_2, P_3. Each product has to pass through two machines M_1 and M_2. Each unit of P_1 requires 3 hours of M_1 and 2 hours of M_2, each unit of P_2 requires 2 hours of M_1 and 5 hours of M_2; and each unit of P_3 requires 2 hours of M_1 and 3 hours of M_2. The machines M_1 and M_2 are available for 30 hours and 40 hours respectively. The profit on each unit of products P_1, P_2 and P_3 is ₹ 4, ₹ 2 and ₹ 3 respectively. If all the manufactured products are sold, formulate the problem as an LPP to maximize the profit.

Unit-II

2. (*a*) Use Dual Simplex method to solve the following :

Maximize $Z = -2x_1 - x_3$

Subject to $x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

and $x_1, \ x_2, \ x_3 \geq 0$

(*b*) Use Big-M method to solve the following :

Maximize $Z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$ and $x_1, x_2 \geq 0$

(*c*) Write the dual of the following L.P. problem :

Minimize $Z_1 = 3x_1 - 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$

Unit-III

3. (*a*) For the following L.P.P

Minimize $Z = \lambda x_1 - \lambda x_2 - x_3 + x_4$

Subject to $3x_1 - 3x_2 - x_3 + x_4 \geq 5$

$2x_1 - 2x_2 + x_3 - x_4 \leq 3$

and $x_1, x_2, x_3, x_4 \geq 0$

find the range of λ over which the solution remain basic feasible and optimal.

(*b*) An office equipment manufacturer procues two kinds of products, chairs and lamps. Production of either a chair or a lamp requires 1 hour of production capacity in the plant. The plant has a maximum capacity of 10 hours per week. The gross margin from the sale of a chair is ₹ 80 and ₹ 40 for that of a lamp. Formulate the problem as a goal programming problem if the goal of the firm is to earn a profit of ₹ 800 per week.

(*c*) Explain the graphical solution to a general programming problem.

Unit-IV

4. (*a*) Solve the following transportation problem in which cell entries represent unit costs :

	To			Available
From	2	7	4	5
	3	3	1	8
	5	4	7	7
	1	6	2	14
Required	7	9	18	34

(*b*) Solve the minimal assignment problem whose effectivenses matrix is given by :

	1	2	3	4
I	2	3	4	5
II	4	5	6	7
III	7	8	9	8
IV	3	5	8	4

(*c*) Prove that a necessary and sufficient condition for the existence of feasible solution of a transportation problem is

$$\sum a_i = \sum b_j \left(i = 1, 2 \ldots m,\ j = 1 \ldots n\right)$$

Unit-V

5. (*a*) Define the following :

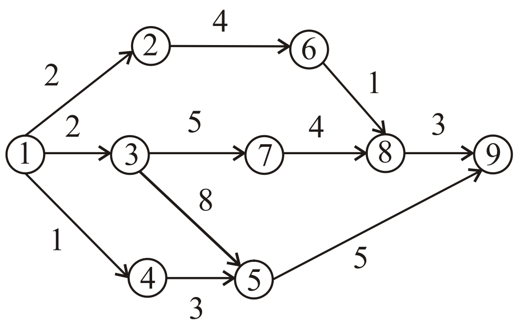
(*i*) Merge event

(*ii*) Burst event

(*iii*) Total float

(*iv*) Free float

(*b*) Find the critical path and calculate the slack time for each event for the following PERT diagram :



(*c*) A project has the following time schedule :

Activity	Time in weeks
(1-2)	4
(1-3)	1
(2-4)	1
(3-4)	1
(3-5)	6
(4-9)	5
(5-6)	4
(5-7)	8
(6-8)	1
(7-8)	2
(8-9)	1
(8-10)	8
(9-10)	7

Construct PERT network and compute :

(*i*) T_E and T_L for each event

(*ii*) Float for each activity

(*iii*) Critical path and its duration

FD-621

M.A/M.Sc. 3rd Semester
Examination, Dec.-Jan., 2021-22

MATHEMATICS

Optional (B)

Paper - V

Graph Theory-I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Prove that if a graph H is homeomorphic from a graph G, then G is a contraction of H.

   (*b*) Prove that any homomorphism is the product of a connected and a discrete homomorphism.

   (*c*) Prove that a graph G is contractible to a graph H and $\Delta(H) \leq 3$. Then G has a subgraph homeomorphism from H.

2. (*a*) Prove that if G is connected and has diameter d then the adjacency algebra has dimension atleast $d + 1$.

(*b*) Prove that any square submatrix of the adjacency matrix F of a graph G has determinant $+1, -1$ or zero.

(*c*) Prove the sum of any two cuts of a graph G is also a cut of G.

3. (*a*) Prove that if a connected k-chromatic graph has exactly one vertex of degree exceeding k-1 then it is minimal.

(*b*) Prove that any uniquely k-colorable graph is (k-1) connected.

(*c*) Prove that every planar graph is 5-vertex colorable.

4. (*a*) Prove that for any graph G, $\alpha_0 + \beta_0 = n$.

(*b*) Prove that for any graph G of order $n \geq 2$ without isolated vertices $\pi_1 \leq \left[\frac{n^2}{4}\right]$ and the partition need use only edges and triangles.

(*c*) Prove that for any connected graph G, $n \geq 2\beta_0 - 1$.

5. (*a*) Prove that a graph is triangulated iff every minimal vertex-separator induces a complete subgraph.

(*b*) Prove that every strongly perfect graph is perfect.

(*c*) Prove that a graph G is a permutation graph iff G and $\bar{G}$ are comparability graphs.