ED-612

M.A./M.Sc. 3rd Semester
Examination, March-April 2021

MATHEMATICS

Paper - I

Integration Theory and Functional Analysis

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Let E be a measurable set such that $0 < \nu E < \infty$. Then there is a positive set A contained in E with $\nu A > 0$.

(*b*) State and prove Radon-Nikodym theorem.

(*c*) State and prove Riesz Representation theorem.

Unit-II

2. (*a*) State and prove Fubini theorem.

(*b*) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ a.e. then f is constant.

(*c*) Let E be a measurable subset of $X \times Y$ such that $\mu \times \nu\,(E)$ is finite. Then for almost all x the set E_x is a measurable subset of Y. The function g defined by $g(x) = \nu\,(E_x)$ is a measurable function defined for almost all x and

$$\int g d\mu = \mu \times \nu(E)$$

Unit-III

3. (*a*) Let K be a compact set, O an open set with $K \subset O$. Then

$$K \subset U \subset H \subset O$$

where U is a r-compact open set and H is a compact G_δ.

(*b*) Let μ be a Baire measure on a locally compact space X and E a r-bounded Baire set in X. Then for $\epsilon > 0$,

(*i*) There is a r-compact open set O with $E \subset O$ and $\mu\,(O \sim E) < \epsilon$.

(*i*) $\mu\, E$ = $\sup\,\{\mu\, k : K \subset E,\ K$ a compact $G_\delta\}$.

(*c*) State and prove Riesz-Markov theorem.

Unit-IV

4. (*a*) Let M be a closed linear subspace of a normed linear space X. Then the quotient space X/M with the norm

$$\| x + M \| = \inf \{\| x + M \| : m \in M\}$$

is a Banach space if X is a Banach space.

(*b*) Let X be a finite dimensional normed linear space. Then any two norms defined on X are equivalent.

(*c*) Show that a bounded linear transformation T from a normed linear space X into Y is continuous.

Unit-V

5. (*a*) Define weak convergence. Let $\{x_n\}$ be a weakly convergent sequence in a normed space X. Then

(*i*) The weak limit of $\{x_n\}$ is unique,

(*i*) The sequence $\{\| x_n \|\}$ is bounded.

(*b*) The dual of l_1 is isometrically isomorphic to l_∞.

(*c*) If X is a finite dimensional normed linear space, then weakly convergent sequence on it is strong convergent.

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MATHEMATICS

Paper - II

Partial Differential Equations and
Mechanics - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove the symmetry of Green's function.

(*b*) State and prove the Poisson formula for half-space.

(*c*) State and prove the mean value formula for Laplace equation.

Unit-II

2. (*a*) Derive the fundamental solution for heat equation.

(*b*) Obtain the solution for $n = 2$ of wave equation by spherical means.

(*c*) State and prove the Euler's-Poisson-Darboux equation for wave equation.

Unit-III

3. (*a*) Derive Kinetic energy in terms of generalized co-ordinates.

(*b*) State and prove the Donkin's theorem.

(*c*) Derive Lagrange's equation of first kind.

Unit-IV

4. (*a*) Prove that the Poisson identity

$$[u, (v, w)] + [v, (w, u)] + [w, (u, v)] = 0$$

(*b*) Find the shortest line on the surface of the sphere.

(*c*) Derive Euler's equation for one dependent variable.

Unit-V

5. (*a*) Find the attraction of a uniform circular disc, of radius a and small thickness k, at a point p on the axis of the disc at a distance p from its centre.

(*b*) Find the potential of a thin uniform spherical shell at any point.

(*c*) Show that a family of right circular cones with common axis and vertex is a possible family of equipotential surfaces and find the potential function.

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MATHEMATICS

Optional (C)

Paper - III

Fuzzy Sets and its Applications - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Let $A_i \in F(x)$ for all $i \in I$, where I is an index set. Then prove that

$$\bigcup_{i \in I} {}^{\alpha-} A_i \leq {}^{\alpha}\left(\bigcup_{i \in I} A_i \right)$$

(*b*) State and prove second decomposition theorem.

(*c*) Prove that the standard Fuzzy intersection is the only idempotent t-norm.

2. (*a*) Prove that a t-norm i and an involute fuzzy complement C, the binary operation u on [0, 1] defined by $u(a, b) = c\ (i\,(a), c\,(b))$ for all $a, b \in [0, 1]$ is a t-conorm such that $<i, u, c>$ is a dual triple.

(*b*) Let A and B are Fuzzy numbers with triangular shape in a Fuzzy equation as

$$A = \begin{cases} 0 & \text{for } x \le 3, x > 5 \\ x - 3 & \text{for } 3 < x \le 4 \\ 5 - x & \text{for } 4 < x \le 5 \end{cases}$$

$$B = \begin{cases} 0 & \text{for } x \le 12, x > 32 \\ (x - 12)/8 & \text{for } 12 < x \le 20 \\ (32 - x)/12 & \text{for } 20 < x \le 32 \end{cases}$$

Find the solution of equation $A \cdot X = B$.

(*c*) Prove that that $< i, u, c >$ be a dual triple. Then prove that the Fuzzy operations i, u, c satisfy the law of excluded middle and the law of contradiction.

3. (*a*) Prove that for Fuzzy sets

$$MIN\ [A,\ MAX\ (A\ B)] = A.$$

(*b*) Let R be a reflexible Fuzzy relation on X^2, where $|X| = n \geq 2$. Then prove that $R_{T_{(i)}} = R^{(n-1)}$.

(*c*) Solve the following Fuzzy relation equation using max-min composition

$$P \circ \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = \begin{bmatrix} .6 & .6 & .5 \end{bmatrix}$$

4. (*a*) If R symmetric, then prove that each power of R is symmetric.

(*b*) Explain Fuzzy compatibility relations.

(*c*) Explain Fuzzy graphs.

5. (*a*) Prove that every possibility measure 'Pos' on a finite power set $P(x)$ is uniquely determined by a possibility distributive function $r : X \to [0, 1]$ via the formula. $\text{pos}(A) = \max r(x)$ for each $A \in P(X)$.

(*b*) Explain the Evidence theory.

(*c*) Let a given finite body of evidence (ε. m) be nested, then prove that for all A, $B \in P(X)$, we have

(*i*) $\text{bel}(A \cap B) = \min[\text{bel}(A), \text{bel}(B)]$

(*ii*) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$

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M.A./M.Sc. 3rd Semester
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MATHEMATICS

Optional - A

Paper - IV

Operations Research - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Use Simplex method to solve the following linear programming problem :

Maximize $z = 6x_1 + 8x_2$

Subject to : $5x_1 + 10x_2 \le 60$

$4x_1 + 4x_2 \le 40$

$x_1, x_2 \ge 0$

(*b*) Solve the following linear programming problem using the result of its dual :

Minimize $z = 24x_1 + 30x_2$

Subject to : $2x_1 + 3x_2 \geq 10$

$4x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 \geq 20$

$x_1,\ x_2 \geq 0$

(*c*) Consider the following linear programming problem :

Maxmize $z = 10x_1 + 15x_2 + 20x_3$

Subject to : $2x_1 + 4x_2 + 6x_3 \leq 24$

$3x_1 + 9x_2 + 6x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$

and check whether the optimality is affected, if the profit coefficients are changed from (10, 15, 20) to (7, 14, 15). If so, find the revised optimum solution.

Unit-II

2. (*a*) Solve the following linear programming problem using big-M method :

Minimize $z = 2x_1 + 3x_2$

Subject to : $x_1 + x_2 \geq 6$

$7x_1 + x_2 \geq 14$

$x_1, \ x_2 \geq 0$

(*b*) Solve the following linear programming problem using dual simplex method :

Minimize $z = 2x_1 + 4x_2$

Subject to : $2x_1 + x_2 \geq 4$

$x_1 + 2x_2 \geq 3$

$2x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

(*c*) Find the Dual of the Primal :

Maximize $z = x_1 + 5x_2 + 3x_2$

Subject to : $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 \geq 0$

Unit-III

3. (*a*) Consider the following parametric linear programming problem :

Maximize $z = (10 - 2t)\, x_1 + (5 - 3t)\, x_2$

Subject to : $8x_1 + 2x_2 \leq 48$

$2x_1 + 4x_2 \leq 24$

$x_1, x_2 \geq 0$

and t is a non-negative parameter. Perform parametric analysis with respect to the objective function coefficients and identify the ranges of t over which the optimality is unaffected.

(*b*) Write a short note on interior point algorithm.

(*c*) Carry out two iterations of Karmarkar's algorithm for the following problem :

Minimize $z = x_1 - 2x_2$

Subject to : $x_1 - 2x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

Unit-IV

4. (*a*) Discuss the similarities and dissimilarities between Transportation and Assignment problem.

(*b*) Use Vogel's approximation method to solve the following transportation problem :

Source \ Destination	1	2	3	4	Supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
Demand	250	350	400	200	

(*c*) Write steps of Hungerian method for solving Assignment problem.

Unit-V

5. (*a*) Write steps of PRIM algorithm for finding the Minimum Spanning Tree problem.

(*b*) A project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers :

Activity (i-j)	Estimate Duration in Weeks		
	Optimistic (t_o)	Most likely (t_m)	Pessimistic (t_p)
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) Draw the project network.

(ii) Find the expected duration and variance for each activity. What is the expected project length ?

(*c*) Consider the following data of the project :

Activity	Predecessor (s)	Duration (weeks)		
		t_o	t_m	t_p
A	–	3	5	8
B	–	6	7	9
C	–	4	5	9
D	*A*	3	5	8
E	*B*	4	6	9
F	*A*	5	8	11
G	*C*, *D*	3	6	9
H	*C*, *D*, *E*	1	2	9

(*i*) Construct the project network.

(*ii*) Find critical path and expected completion time.

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MATHEMATICS

Optional (B)

Paper - V

Graph Theory - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Prove that if a graph H is Homeomorphic from a graph G, then G is a contraction of H.

(*b*) Prove that if G is a k-regular graph, k is an eigenvalue of G. Then this is simple if G is connected. Every other eigenvalue has absolute value $\leq k$.

(*c*) Define the following terms with an example :

(*i*) Direct sum

(*i*) Direct product

(*iii*) Derived graph

(*iv*) Vector space

2. (*a*) Prove that the incidence matrix F of a graph G has rank n-k, where k is the number of components.

(*b*) Prove that any square submatrix of the adjacency matrix F of a graph G has determinant + 1, – 1 or zero.

(*c*) Prove that the sum of any two cuts of a graph G is also a cut of G.

3. (*a*) Prove that if G is a critical graph, then $\delta (G) \geq k - 1$.

(*b*) Prove that any uniquely k-colourable graph is $(k - 1)$ connected.

(*c*) Prove that for any graph G with order $n \geq 4$, $l (G) \leq [n^2/4]$.

4. (*a*) Prove that for any graph G, $\alpha_0 + \beta_0 = n$.

(*b*) Prove that for any graph G, $c (G) = p (G)$.

(*c*) Prove that for any graph G of order $n \geq 2$ without isolated vertices, $\pi_i \leq [n^2/4]$ and the partition need use only edge and triangles.

5. (*a*) Prove that a graph is triangulated iff every mimimal vertex separator induces a complete subgraph.

(*b*) Prove that every comparability graph is perfect.

(*c*) Prove that a graph G is a permutation graph iff G and $\overline{G}$ are comparability graphs.

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